

from the water in which they have been plunged, as soon as the redness in the centre of the drop ceases to be visible.

Since the smallest portion of any polarizing crystal polarizes or depolarizes light according to its position, the author expected to find the same property in the fragments of a broken drop, but upon trial they did not appear to possess this property.

Of the many important conclusions to which the author thinks that these experiments are calculated to conduct us, there is one which he considers too palpable to be passed over, namely, that when the particles of glass are separated to a certain distance by the expansive agency of heat, they assume a crystalline arrangement, which would not be discovered but by fixing them in this state by sudden cooling; since the gradual approximation of the particles, by slow cooling, entirely destroys the crystalline structure thus produced.

In a note the author remarks, that on more than one authority steel is said to be less dense after being hardened by quenching than before, which he ascribes, as in glass, to the sudden induration having commenced at the surface. And he takes occasion to suggest the possibility, that under these circumstances moderate changes of temperature may not occasion any degree of expansion, and that we may obtain, within certain limits, a substance of invariable length that may be useful for pendulums.

Description of a new Instrument for performing mechanically the Involution and Evolution of Numbers. By Peter M. Roget, M.D. Communicated by William Hyde Wollaston, M.D. Sec. R.S. Read November 17, 1814. [*Phil. Trans.* 1815, p. 9.]

The present instrument depends upon a new extension of the principle of the common sliding-rule; for as in that numbers themselves are multiplied or divided by the mechanical addition of their logarithms, so in this their logarithms are multiplied or divided by mechanical application of corresponding logometric spaces.

In the common tables of logarithms, that of 10 is 1, and those of its simple powers are 2, 3, 4, &c.; so also the logarithm of the square root of 10 is $\frac{1}{2}$, or .5; the fourth root is $\frac{1}{4}$, or .25, being a decimal index expressing a power of 10 less than unity. In the same manner all other numbers are considered as powers of 10, and their logarithms are integral or decimal indices of those powers.

In the common sliding-rules the divisions are so placed as to mark intervals that are proportional to these indices; so that by simple juxtaposition the sum or difference of any two indices, and consequently the product or quotient of any two numbers, appears by inspection.

In this manner, by addition of two equal logometric intervals, the square of any number may be found; but the instrument so constructed is not prepared to give the higher powers, without proportionally frequent repetitions of the same process, which gives at length a multiple of the index by the tedious operations of repeated addition.

The instrument contrived by Dr. Roget, is constructed to answer this last purpose, with the same facility as common multiplication and division are performed by the common sliding-rule.

For in the same manner as numbers are considered as powers of 10, so their indices, whether integral or decimal, being, in fact, numbers, may again be regarded as powers of 10; and their secondary indices, or logarithmic logarithms, may be laid down as logometric intervals, to which other logometric intervals may be added or subtracted mechanically, so as to present to view any multiples or aliquot part of a logarithm, and consequently any powers whatever of the number to which that logarithm is index; for when the unit of Gunter's line, on the slider, is applied to any number on the scale, divided into logometric logarithms, then 2 on the slider corresponds to the square, 3 to the cube, &c. of the same number.

The author enumerates various uses to which such an instrument is applicable. To all cases of geometrical progression the application is obvious for finding the common ratio, the number of terms, or any particular term in the series. An approximate solution is thus given to all questions of compound interest, to regularly progressive increase of population, and to many calculations of chances. To cases also of the reduction of temperature which a body undergoes by communication to a surrounding medium; and to successive stages of exhaustion, by an air-pump, it may be applied with equal advantage.

Since the scale of the instrument presents to view the proportion of logarithms to each other, while the slider represents the ratios of their respective numbers, it becomes a means of illustrating many points relative to the general theory of logarithms, whether to exhibit a series of logarithms formed according to any particular modulus, or by an inverted position of the slider to find the moduli of all different systems, and accordingly in that, for instance, in which the modulus is equal to the basis.

In this inverted position of the slider it affords a ready answer to various exponential equations, which do not admit of any direct solution, as, for instance, if $x^x = 100$. Let the unit on the slider be placed opposite to 100 on the rule, then 2 will be found opposite to 10, which is the square root of 100; 3 will be opposite to 4.641, which is the third root of 100; and by attending to the decreasing numbers which correspond to increasing numbers on the slider, it will be seen that 3.6 is the point at which they appear equal, showing that $3.6^{3.6}$ is nearly equal to 100, and is therefore approximately the root required.

The author concludes by pointing out various forms in which such an instrument may be constructed; since the line divided into logometric logarithms may be turned into a spiral, or arranged according to any other of the various modifications that have been given to the common logometric line of Gunter.